STUDY OF THE WATER-FILLED CYLINDRICAL ACOUSTICAL RESONATOR FOR THE DETERMINATION OF LOW FREQUENCY ATTENUATION IN SEA WATER.

by

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THESIS

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for the Determination of
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ABSTRACT

A pair of large cylindrical Pyrex vessels were water-filled to various heights and the acoustic damping constants and resonant frequencies determined from 3 to 15 kHz. Evacuation of a chamber surrounding the cylinder and increase of the ratio of water height to vessel radius, h/R , greater than unity are both shown to reduce the relative ambient loss of the resonant system. Although the highest Q (20,800) is much less than attained with spherical vessels, it is believed that with thinner walled vessels the extrapolation method of accounting for the influence of the boundaries will permit laboratory measurement of the low frequency attenuation of sound in sea water.

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ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Dr. H. Medwin for his guidance and support in this investigation.

I. INTRODUCTION

Attenuation of sound in water has been a subject of interest for more than thirty years. Early experiments —1827— were conducted on Lake of Geneva by Colladon and Sturm [1]. Many workers have since studied the absorption of sound in fluids. An excellent paper on the subject, which has an extensive list of references, has been written by J. J. Markham et al. [2].

In the laboratory, there are several different methods for measuring the attenuation of sound in liquids. In each of the methods, the attenuation is easier to measure at higher frequencies because the attenuation increases with frequency. Recent, authoritative, data were obtained by R. W. Leonard who introduced a resonator method [3] to measure the attenuation of sound in water, sea water, and synthetic sea water at frequencies between 50 and 350 kHz.

Leonard sonically excited a liquid-filled spherical container with single frequencies and measured the decay rate for a large number of these excitations. The decay rate can be expressed in terms of a temporal absorption coefficient κ . The spatial absorption coefficient, α , is found from the temporal absorption coefficient by dividing by the velocity of sound in the liquid. The total experimental rate of energy decay depends on the attenuation in the liquid and the attenuation due to the container and its associated fittings; the latter attenuation being called losses for brevity. How small the losses can be made is the critical factor in determining how small an attenuation can be measured with reasonable accuracy. Because of the very low attenuation of low frequency sound in water, to make successful measurements below 10 kHz it is particularly necessary to minimize losses. In addition to minimizing losses due to material, structural and electronic fittings, the driving frequency should be selected to excite a radial mode of oscillation since the pure radial modes, in a perfect pressure-release sphere, have no shear viscous attenuation. The radial modes are only a small fraction of the possible modes of excitation. Most of the modes have a tangential velocity component at the walls and thus have shear viscous losses associated with these sloshing modes which cause the apparent decay rate to increase.

For a given container and a radial mode of excitation, this method is used to compare decay rates of various liquids against the decay rate of some standard, with the result ordinarily being stated as a relative absorption coefficient.

One of the practical limitations of the system is that the glass spheres are usually boiling flasks which are blown and are not precisely spherical. The imperfections cause the excited mode to be sloshing to some degree, with the associated excess losses. Nevertheless, spherical containers have been used successfully by Wilson and Liebermann [4], Kurtze and Tamm [5], Karpovich [6], and others for sound attenuation measurements in water at frequencies over 20 kHz.

Cylindrical containers have been used by Mulders [7] and others to determine the attenuation of sound in water in the vicinity of 1 MHz by measuring the decay time. By using an extrapolation technique, a series of tests with different volumes — and surface areas — of a liquid gives results from which the separation of absorption in the liquid and absorption in the container can be accomplished.

Since a glass cylinder can be formed more precisely to a specified diameter than a glass sphere [8], and since it appears that attenuation can be separated from extraneous losses, we have sought to combine the two methods to advantage.

The object of this research can be stated as follows: to optimize the parameters of a cylindrical resonator for the laboratory measurement of low frequency sound attenuation in sea water.

II. THEORY

In order to have some knowledge of what resonant frequencies can be expected from a cylindrical vessel of radius a, filled to a height h, it is necessary to look at the steady state wave equation in cylindrical coordinates. The solutions — eigenfrequencies — will be displayed in tabular form for the first few modes of oscillation.

A. EIGENFREQUENCIES OF CYLINDRICAL VESSEL

Assuming the pressure P in the vessel varies with radius, height of the water, and angular displacement, and that these variations are not inter-related one can write

$$P = R(r)\, Z(z)\, \Phi(\phi)\, e^{j\omega t} \tag{1}$$

Since a steady state solution is desired, the time variation is of no interest. Substituting (1) into the wave equation

$$\nabla^2 P = \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2} \tag{2}$$

we obtain

$$\nabla^2 P = -k^2 R Z \Phi \qquad \text{where} \quad k^2 = \frac{\omega^2}{c^2} .$$

Separating the variables gives

$$\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = -k_z^{\,2} \tag{3}$$

and

$$\frac{1}{R} \frac{\partial}{\partial r} \left(r \frac{\partial R}{\partial r}\right) + \frac{1}{r^2 \Phi} \frac{\partial^2 \Phi}{\partial \phi^2} = -k_r^{\,2} \tag{4}$$

where $$k^2 = k_z^2 + k_r^2 \qquad \text{and is constant.} \tag{5}$$

A solution for (3) is

$$Z = A \cos k_z a + B \sin k_z z \tag{6}$$

Equation (4) can be written as

$$\frac{r}{R} \frac{\partial}{\partial r} (r \frac{\partial R}{\partial r}) + r^2 k_r^2 + \frac{\partial^2 \Phi}{\partial \phi^2} = 0 .$$

Then separating variables again

$$\frac{\partial^2 \Phi}{\partial \phi^2} = -m^2 \tag{7}$$

and

$$\Phi = C \cos m\phi + D \sin m\phi \tag{8}$$

where m^2 is a constant. Equation (4) also gives

$$\frac{r}{R} \frac{\partial}{\partial r} (r \frac{\partial R}{\partial r}) + r^2 k_r^2 = m^2$$

which can be written in the form of Bessel's equation, i.e.,

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} + R (k_r^2 - \frac{m^2}{r^2}) \tag{9}$$

Then

$$R = J_m(k_r r) \quad . \tag{10}$$

Now Equations (6), (8), and (10) must all satisfy the boundary conditions. For the cylinder investigated here, the walls will be thin — small with respect to the wavelength of the excitation. Therefore, all surfaces will be assumed pressure-release. Then

$$A = 0 \quad , \quad J_m(k_r a) = 0 \quad .$$

D can be set equal to zero because it depends only on the orientation of the cylinder with respect to the origin of ϕ .

From the above, P can be written

$$P = p_o(\sin k_z z)(\cos m\phi)\, J_m(k_r r) \tag{11}$$

where $p_o = BC$. Using the assumed pressure release at the surface,

$$\sin k_z z = 0 \qquad \text{when} \quad k_z z = \ell\pi \ , \quad \ell = 0,1,2, \ldots$$

or

$$k_z = \frac{\ell\pi}{h} \ , \quad \ell = 0,1,2 \ldots \qquad \text{and} \qquad f_z = \frac{\ell c}{2h} \tag{12}$$

since

$$k = \frac{2\pi f}{c} \quad .$$

With radial symmetry, m equals zero and P equals zero when $J_o(k_r r)$ equals zero. This occurs at

$$k_r a = 2.405,\ 5.520,\ 8.659,\ \ldots \tag{13}$$

For notational simplicity let $k_r = k_{mn}$. Then

$$k_{01} = \frac{2.405}{a} \ , \quad k_{02} = \frac{5.520}{a} \ , \quad \ldots \ , \quad \ldots$$

With $m = 1$, $J_1(k_r a) = 0$ and this occurs at

$$k_{11} = \frac{3.85}{a} \ , \quad \frac{7.02}{a} \ , \quad \ldots$$

Equation (5) can be written as

$$f = \sqrt{f_{\ell}^{2} + f_{m,n}^{2}} \tag{14}$$

which gives the system theoretical resonant frequencies. Temperature variations [9] can be considered using the following formula

$$f_{\ell,m,n} = \frac{1403 + 5T - 0.06T^{2} + 0.0003T^{3}}{2\pi \times 10^{-2}} \sqrt{k_{\ell}^{2} + k_{mn}^{2}} \tag{15}$$

where T is in degrees Centigrade and the dimensions are in centimeters.

Table I shows the theoretical resonant frequencies calculated for cylinders of radius 8.4 and 14.3 cm, both with a water depth of 10.0 cm. Sound velocity was assumed to be 1500 m/sec.

B. SEPARATION OF ABSORPTIONS

In an unbounded vessel filled with liquid the acoustic energy in the liquid would decay, after the source of energy has been removed, as a function of the liquid and time according to

$$E = E_0 \, e^{-2\kappa t} \tag{16}$$

where κ is the temporal damping constant, E is the total energy in the system and E_0 is the total energy in the system the moment the source is removed.

In a bounded vessel there will be energy losses due to the following factors:

1. absorption in the liquid
2. absorption in the material of the vessel and along the liquid-to-material interface
3. energy radiated into the air and conducted by the vessel mount and fittings.

cm		m	0	0	0	0	1	1	1	2	2	2
a	ℓ	n	0	1	2	3	1	2	3	1	2	3
8.4	0		—	6.80	15.6	24.4	10.9	20.0	28.9	14.6	24.0	33.0
8.4	1		7.5	10.2	17.3	25.6	13.2	21.4	29.9	16.4	25.2	33.8
8.4	2		15.0	16.4	21.6	28.9	21.0	25.0	33.6	20.9	28.4	36.2
8.4	3		22.5	23.6	27.4	33.4	25.0	31.7	36.7	28.6	33.0	39.9
14.3	0		—	4.08	9.18	14.4	6.42	11.7	16.8	8.57	14.1	19.4
14.3	1		7.5	8.52	11.7	16.2	9.87	13.9	18.4	11.4	16.0	20.7
14.3	2		15.0	15.5	17.5	20.8	16.3	19.0	23.2	17.3	20.6	24.6
14.3	3		22.5	22.9	24.3	26.8	23.5	25.4	28.2	24.1	26.6	29.8

Table I. Theoretical System Resonant Frequencies, $f_{\ell,m,n}$ (kHz)

Energy decay, under the same circumstances as in equation (16), for the bounded vessel is

$$E_t = E_0 e^{-2\delta t} \tag{17}$$

where δ is the temporal damping factor for the total system and E_t is the total energy in the system. The total energy in the system is the energy in the liquid plus the energy in the vessel itself. The walls and the liquid are a coupled acoustic system and the total energy in the system will decay with one time constant, $\frac{2}{\delta}$. Here we will define $T = \frac{1}{\delta}$ to be the time for the acoustic pressure to decay to 1/e times its original value.

Assuming excitation of a pure radial mode in the cylindrical vessel there is an average per cm^3 in the liquid, $\underline{E}$, and the energy in the walls of the container is proportional to $\underline{E}$. Therefore,

$$E_t = \underline{E}V + \underline{E}S \tag{18}$$

where V is the volume of the liquid and S is a constant of proportionality. If the rate of energy decay is slower in the liquid than in the walls, then there is a continuous flow of energy from the liquid to the walls.

The part of the incremental change in energy per unit time due to absorption in the liquid is

$$\Delta E_\ell = 2\kappa \underline{E}V \tag{19}$$

from differentiation of equation (16).

The rate of loss in the part of the side walls covered by water is

$$\Delta E_W = 2w\underline{E}\, 2\pi Rh \tag{20}$$

where h is the height of the water, R is the radius of the vessel and w is a function of the material, frequency, and wall thickness, but not of h or R, and is a constant of proportionality.

For the bottom the rate of loss is

$$\Delta E_b = 2b\underline{E}A \tag{21}$$

where A is the area of the bottom, and b is a constant of proportionality. Equation (21) can also include the losses at the upper surface of the liquid, if any.

The sound energy in the vessel walls above the water is also proportional to $\underline{E}$ since there is no discontinuity in the walls of the vessel and the distribution of energy is again uniform. Then in the walls above the liquid,

$$\Delta E_u = 2u\underline{E}\, 2\pi R(H-h) \tag{22}$$

where H is the total height of the vessel. Then, u is a constant of proportionality associated with the losses in the walls above the liquid and is not a function of h or R.

From the mount and fittings, the rate of loss is

$$\Delta E_F = 2F\underline{E} \tag{23}$$

where F is a proportionality constant which depends on the method of attachment, frequency, and material but does not depend on h.

Combining all of the above effects,

$$\frac{d\ E_t}{dt} = 2\underline{E}\ \{\kappa Ah + w2\pi Rh + bA + u2\pi R(H-h) + F\} \tag{24}$$

which gives

$$\delta = \frac{\underline{E}A \left[\kappa + \frac{2\pi R(w-u)}{A}\right] h + b + \frac{F}{A} + \frac{u2\pi RH}{A}}{\underline{E}Ah + \underline{E}S} \tag{25}$$

when substituted into equation (17). For large volumes and thin walls the second term in the denominator can be neglected, and therefore,

$$\delta h \approx \left(b + \frac{F}{A} + \frac{u2\pi RH}{A}\right) + \left(\kappa + \frac{2\pi R(w-u)}{A}\right) h \tag{26a}$$

or

$$\delta h \approx C + Dh \tag{26b}$$

where

$$C = b + \frac{F}{A} + \frac{uH2\pi R}{A} ,$$

$$D = \kappa + \frac{2(w-u)}{R} . \tag{26c}$$

C is a measure of the losses in the bottom fittings and radiation loss and is a function of frequency. D is a measure of the losses in the walls of the vessel and radiation loss as well as the absorption in the liquid.

By comparing measurements of T in similar vessels at the same frequency and temperature and keeping the factor $2(w-u)$ constant, D can be determined by a variation of h in each of two or more similar vessels. Since the factor which is to remain constant is related to the side walls of the vessel, the walls must be of the same material and of the same thickness. The bottom may be different if necessary.

C. PARAMETERS TO BE OPTIMIZED

As has been indicated previously, since the low frequency sound absorption in water is so small, the attenuation caused by anything other than the liquid itself must be minimized. The factors to be considered can be listed as:

1. the cylinder material
2. geometry of the cylinder
3. radiated and conducted losses
4. mode excitation.

1. Material

Absorption properties of some materials are listed by Mason [10] in terms of a mechanical Q factor, where Q is the ratio of energy stored to energy dissipated, and in terms of heat flow [11] for several of the same materials. One minimizes losses due to material by selecting a material of high Q and low heat flow. Fused silica, Q=5,000 , appears to be a very desirable material, but due to difficulties in forming large cylinders, and very high cost, fused silica was rejected. Pyrex with a Q of 1,200 is more easily formed, has low heat flow, is inert chemically and is less expensive than fused silica. Due to the above considerations Pyrex was selected, although aluminum appears to be a good choice due to its very high Q of approximately 10,000.

2. Geometry of the Cylinder

From the quality factor of 1,200 and the velocity of sound in the Pyrex, one can determine a spatial absorption coefficient [12]. To get an order of magnitude estimate of maximum ratio wall thickness to radius, we set the losses in the side walls equal to the attenuation in the liquid. If this is done and plane waves are assumed, the wall thickness to radius ratio of approximately 0.005 is obtained for sea water, and would require a wall thickness of less than 1.0 mm for a cylinder resonant at about 5.0 kHz. The ratio for fresh water is approximately 0.0005.

The height to radius ratio should be as large as reasonably possible since shear losses associated with the bottom are a small part of the total attenuation if the volume of liquid is large.

3. Radiation and Conduction Losses

Both C and D of equation (26c) are functions of u, the constant of proportionality for the radiation losses. Radiation through the air can be minimized by evacuating the space surrounding the cylinder.

Conduction losses may also be significant. These losses can be reduced by minimizing the supporting material, and avoiding resonances in the mounting or suspension structure in the vicinity of the frequencies of interest.

4. Mode Excitation

Since theoretically the minimum system loss will occur for a pure radial mode, it is desirable to excite a pure radial mode in order to have a large signal to noise ratio. This can be done by driving at a position of theoretical maximum pressure for a pure radial mode at the proper frequency.

III. EQUIPMENT

The equipment used in this investigation consisted of a vacuum system, to minimize radiation losses, and an electronic system, to excite resonance and to display the decay of sonic energy.

A. MECHANICAL

The mechanical vacuum system consists of an aluminum cylinder to enclose the resonator, a vacuum pump, and a water trap. The cylinder was made of 0.25 inch aluminum, is 76.0 cm by 59.0 cm, inside diameter, and has a removable 1.0 inch aluminum top plate. The Welch Duo Seal vacuum pump is connected to the cylinder through a glass-liquid nitrogen water trap.

B. ELECTRONIC

A block diagram of the electronic system is shown in Figure 1. The General Radio Coherent Frequency Synthesizer provides a highly accurate frequency source, which is gated by a General Radio Type 1396A Tone Burst Generator. The gating signal is provided by a Hewlett Packard 200A Audio oscillator. The exciting frequency is on for approximately 100 msec and is off for a variable time up to about 10 seconds. When the tone burst generator is off, its output is 44 dB below the input level. This was not sufficient isolation. Therefore an external gate, as shown in the circuit diagram in Figure 2, was inserted to provide another 40 dB of isolation. The pulsed signal is then fed through an H. H. Scott Model 280 power amplifier and impedance matching transformer to a cylindrical ceramic transducer mounted co-axially on the bottom of the Pyrex cylinder, which sonically excites the resonant system.

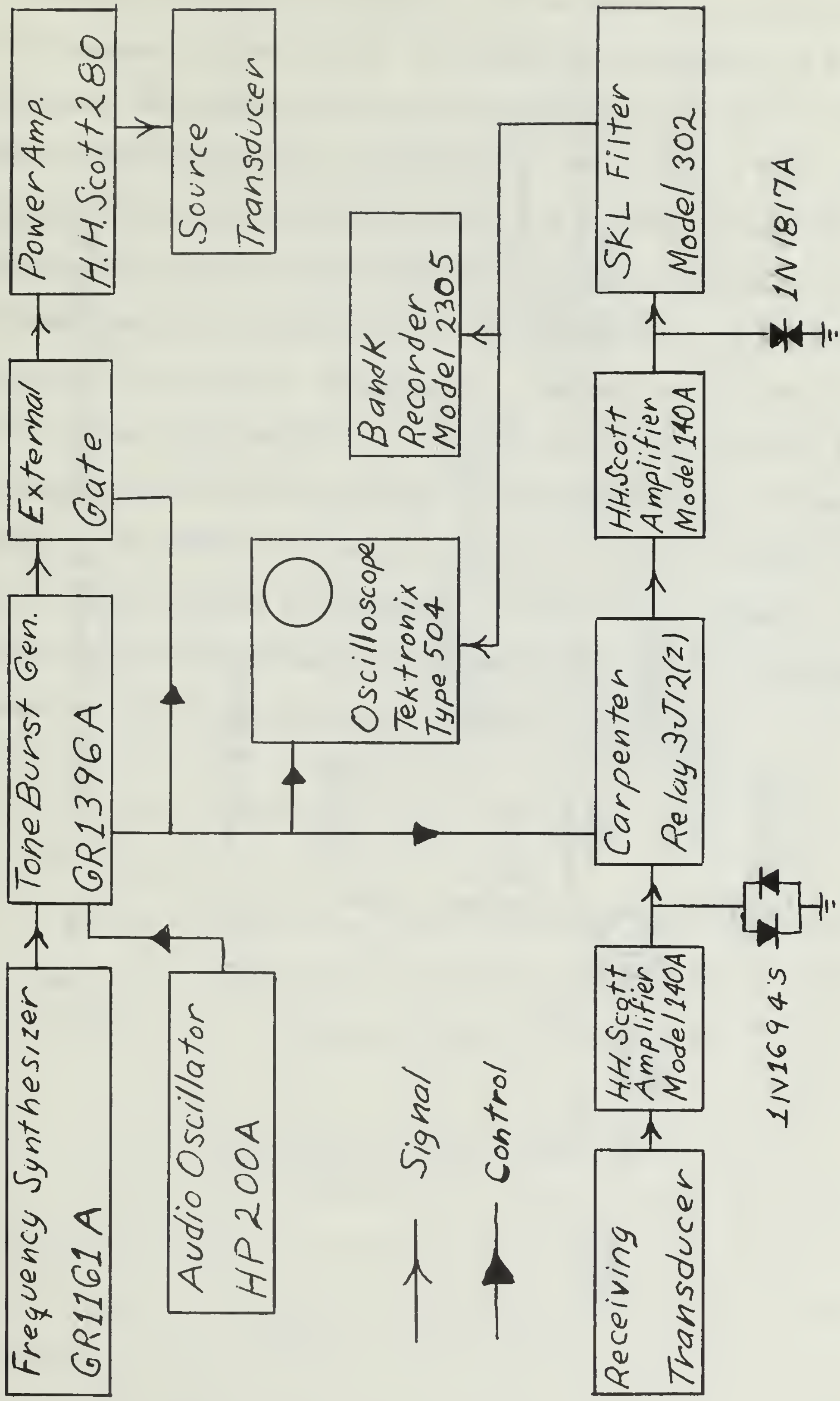
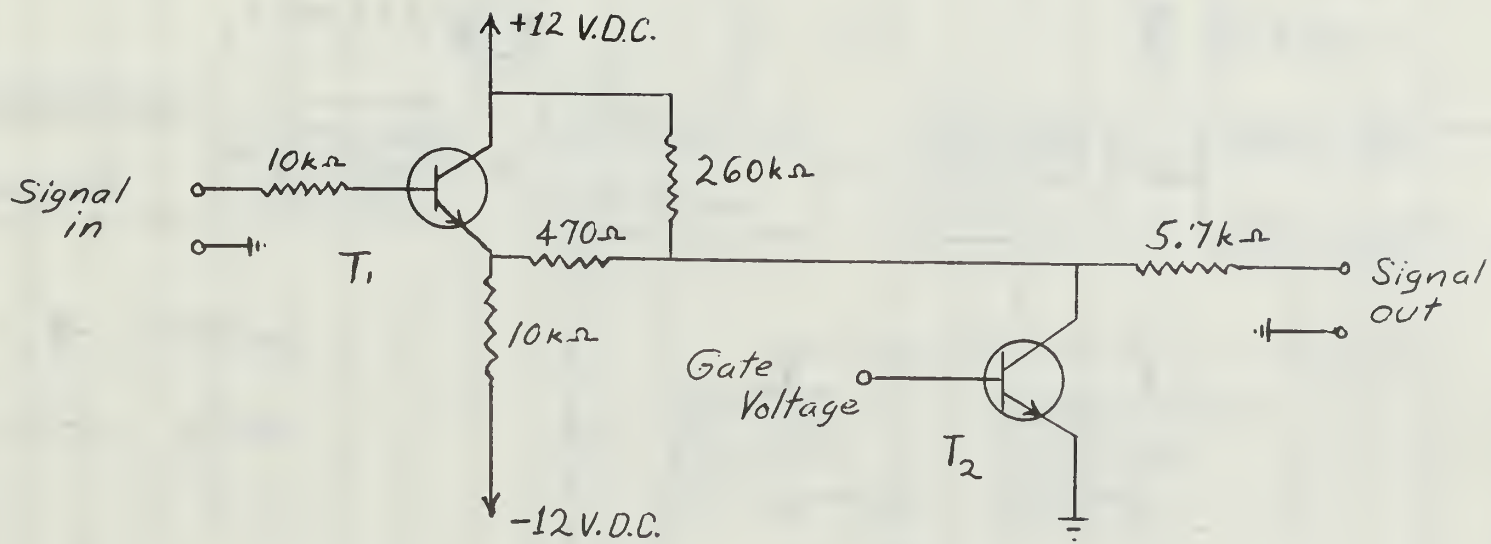


Figure 1 System Block Diagram



T_1, T_2 2N736'S

Fig. 2 External Gate Circuit Diagram

A smaller cylindrical transducer is also mounted co-axially to act as a microphone for the system. Its output is amplified by an H. H. Scott Model 140A decade amplifier and then limited by a pair of 1N1694 diodes connected as a full wave rectifier. The signal is then passed through a Carpenter Type 3J12(z) relay. The function of the relay is to short the received signal to ground during the period of excitation to prevent amplifier saturation since the energy after the excitation has ceased is the area of investigation. The desired portion of the signal is then again amplified and limited at a higher level by a Texas Instruments 1N1817A limiting diode. The amplified signal is then passed through an SKL Model 302 Variable Electronic Filter set to have a band pass of 1 kHz, centered approximately at the exciting frequency. The voltage decay is then viewed on a Tektronix Model 504 oscilloscope and recorded on a Bruel and Kjaer Level Recorder Type 2305.

IV. EXPERIMENTAL RESULTS

A. PROCEDURE

A portion of the experimental results has already been stated. The final coaxial ceramic transducer excitation system was the result of prior comparison of frequency response curves with theoretical radial frequencies and an examination of acoustic pressure variations in the radial and axial directions in the vessel. In the preliminary studies, both single and multiple transducers were placed in a variety of positions on the bottom and along the sides of the vessel at various heights and angles with respect to each other in a search for the optimum excitation of the radial modes. Source location on the side of the vessel excited an originally — unexpected, presumably flexural — low resonant frequency of the container itself.

Measurements of decay times were taken in two cylindrical vessels for various depths of water. Vessel No. 1, of cast Pyrex, had an inside radius of 14.3 ±0.2 cm and a wall thickness 9.0 ±0.8 mm. Vessel No. 2, constructed of extruded Pyrex tubing with fused bottom, had an inside radius of 8.4 ±0.1 cm measured at various axial and azimuthal positions, and a quite uniform wall thickness of 3.65 ±0.10 mm. Both vessels were 60 cm high.

The water sample used in the cylinders was distilled water that had been placed under a vacuum of approximately 29.5 inch Hg for a period of at least 8 hours. After 10 hours under this vacuum the decay time no longer increased and it was assumed the sample was degassed. In order to keep the sample degassed it was necessary to keep it under a vacuum. One large sample of distilled water, 72 liters, was used throughout the experiment.

Figure 3a shows a typical oscilloscope display of the acoustic pressure decay in the vessel. Figure 3b shows a less typical decay with an interference pattern. These interference patterns are caused by the interactions between the sound reflected from the inner boundary of the wall and from the outer boundary. This type of pattern has been avoided. Figure 3c is a tracing of the decay shown in Figure 3a and is included to show how the decay time was obtained from the oscilloscope display.

Figure 4 is a typical record from the B and K level recorder. Since the record is calibrated in dB, the slope of the curve is proportional to the system time constant. Whem measuring the decay time, an average of 5 to 6 readings taken from the B and K recorder was compared against a reading taken directly from the oscilloscope. If the comparison seemed reasonable, the readings were recorded without rechecking them. If the comparison was poor, the readings were retaken and rerecorded until there was a reasonable comparison between the two methods of determining the decay time.

For example, by looking at Figure 3c one can see that the decay time is on the order of 70 msec. From Figure 4, the band K record, using the known scale factor, chart speed, and the formula

$$T = \frac{8.68\ (\text{dB/neper})\ (\#\text{mm}/10\ \text{dB})}{\text{chart speed in mm/sec}} ,$$

one obtains in this case

$$T = \frac{(8.68)\ (7.6)}{100} = 65.6\ \text{msec}$$

which is a reasonable comparison.

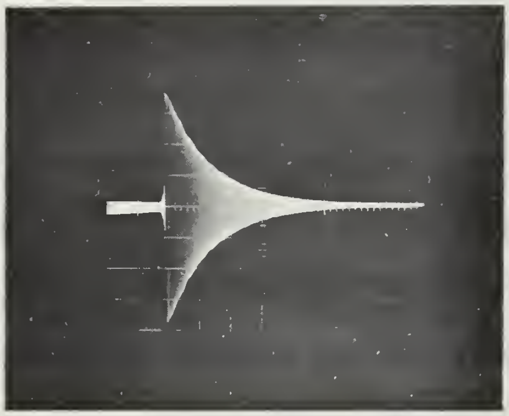


Figure 3a
Typical
Oscilloscope
Display

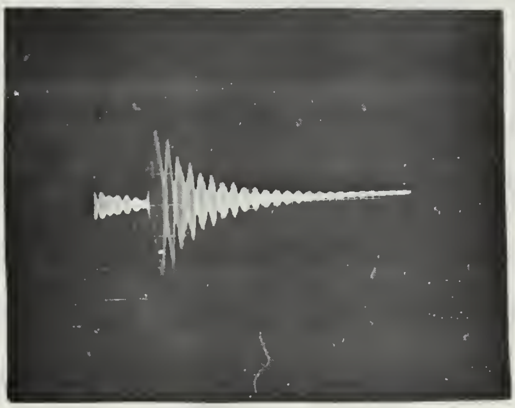


Figure 3b
Oscilloscope
Display with
Interference
Pattern

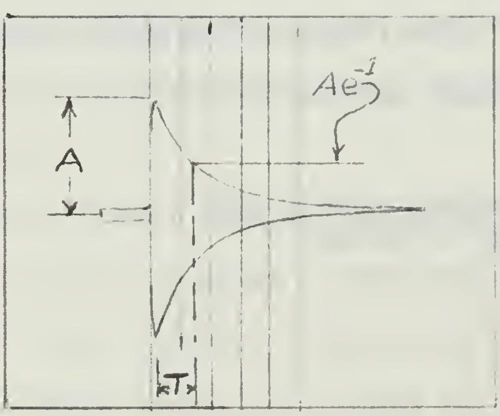


Figure 3c
Determination
of Decay Time,
T, Directly

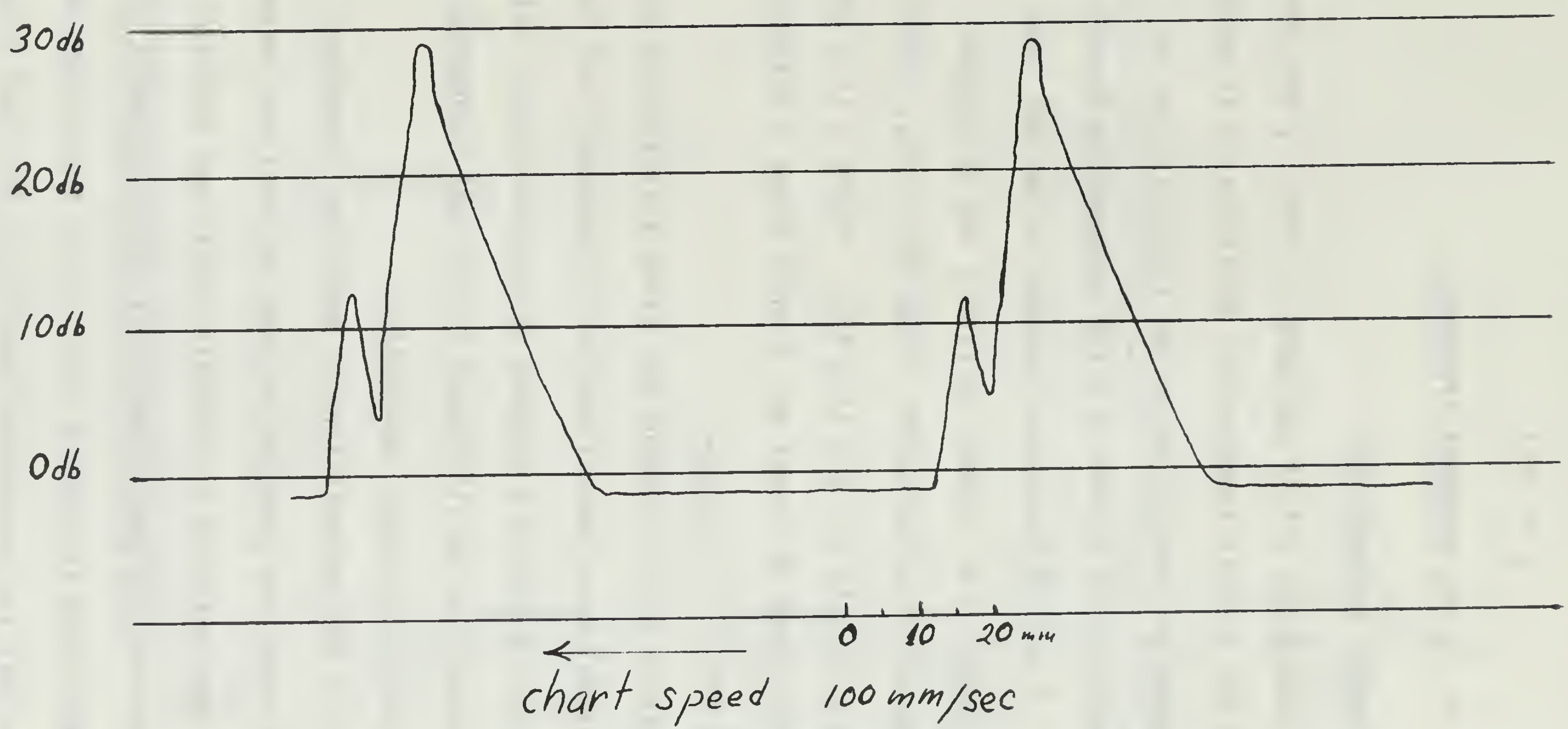


Figure 4 Typical Record from B & K Recorder

Decay time and Q are related by

$$Q = f\pi T$$

where f is the resonant frequency.

B. RESONANT FREQUENCIES

A determined effort was made to identify the precise mode of oscillation for several frequencies, particularly in vessel No. 2. A 1/8 inch hydrophone was used to probe the liquid in the radial and axial directions. Frequencies of 6.93 and 15.6 kHz appeared to be very nearly pure radial modes. A set of frequency response curves was taken with variations in water depth in an attempt to find a set of resonant frequencies that were independent of water depth. From the curves, radial modes appear to be at approximately 6.8 and 16.2 kHz. Figure 5 is a typical frequency response curve of vessel No. 2 which shows a resonance in the vicinity of 6.8 kHz.

Neither of the methods has given satisfactory results as far as unequivocal mode identification is concerned, but the experimental frequencies are within 5 percent of the theoretical values. The frequency discrepancy can be attributed to the wall boundaries not agreeing with the "pressure release" assumption.

Since the determination of modes was imprecise, and since the pure radial modes are expected to show the smallest losses, the longest decay times were selected as probably radial and interest was focused on these. In addition, the requirement that the measurement frequency be independent of water depth (to assure that it is a pure radial mode) and the desirability that the measurement mode be of the same frequency for both vessels made 8.3 kHz the particular frequency of greatest interest.

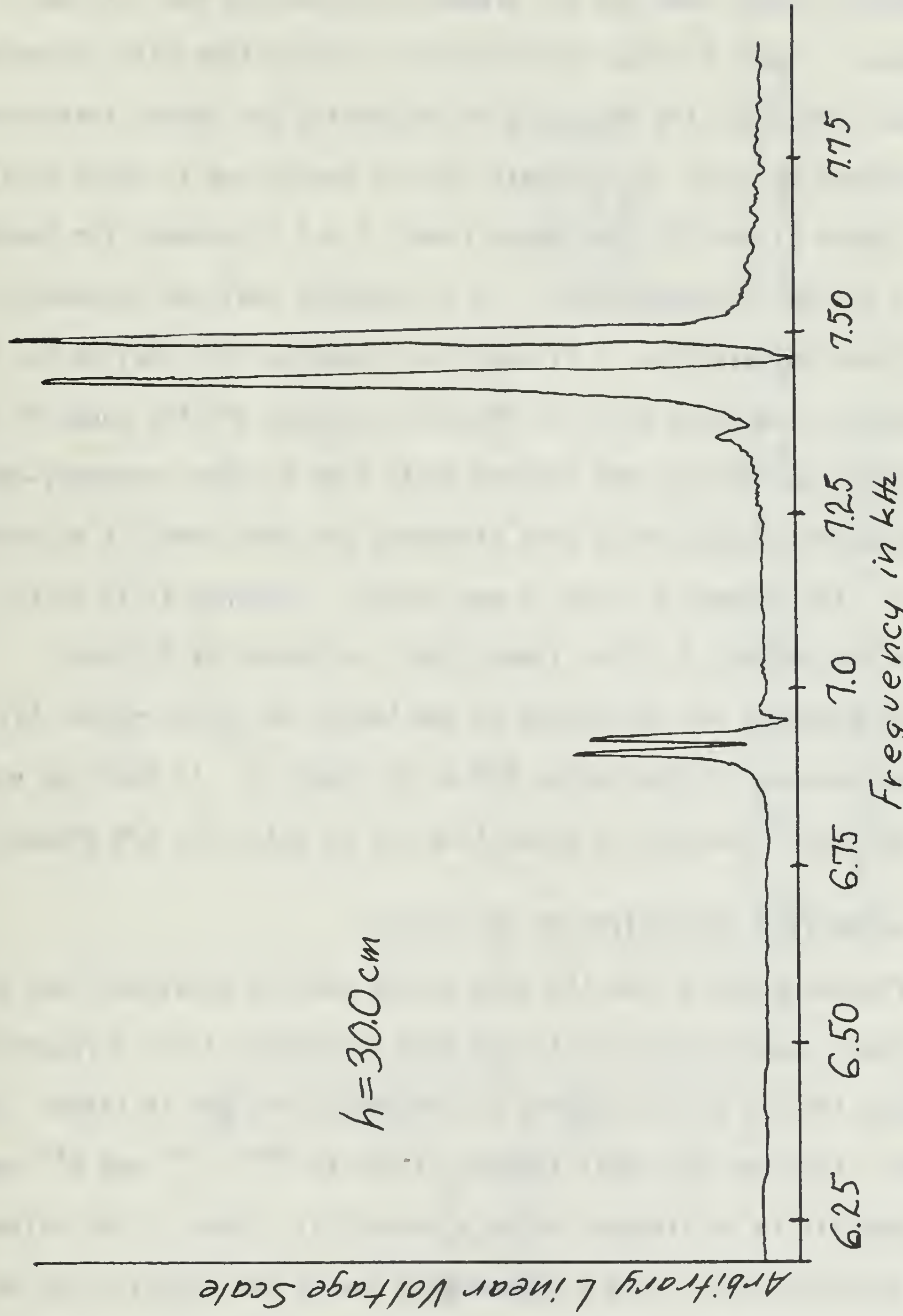


Figure 5 Frequency Response Curve, Vessel 2.

C. VARIATIONS IN DECAY TIME

Measurements were taken in vessel No. 1 to show the decrease in radiation losses when the air pressure surrounding the cylinder is reduced. Figure 6 shows the increases in decay time with increased vacuum, and shows the necessity of evacuating the region surrounding the vessel in order to eliminate loss of energy due to sound radiation.

Tables II and III show decay time, $T = \frac{1}{\delta}$, in msecs for vessels No. 1 and No. 2 respectively. It is apparent that the information available on vessel No. 2 is much more complete than that on No. 1. Evidently, the thick walls of the cast cylinder are the cause of the inability to identify and isolate decay time at many frequency-depth combinations. Supporting this statement are the lower Q's of vessel No. 1. The highest Q of No. 2 was 20,800 , obtained at 14.75 kHz, while the highest Q of No. 1 was 5,500 , obtained at 8.3 kHz.

In order to see the effect of the height to radius ratio, h/R, the decay time was plotted versus h/R as in Figure 7. As could be expected, a significant increase in decay time can be noted for h/R greater than 1.

D. ABSORPTION COEFFICIENT OF THE LIQUID

Figures 8 and 9 show the data points and the straight lines that are the least squares best fit to the data of vessels 1 and 2 respectively. Because the sum of the squares of the deviations was so large — about 4,000 — the data was least squares fitted to 2^{nd}, 3^{rd}, and 4^{th} order polynomials in an attempt to get a better fit. None of the polynomials fit the data well. This is presumably due to the inability to maintain a fixed mode as height of the water was increased.

According to the theory, the slopes of the straight lines in Figures 8 and 9 are the D of equation (26). When the value of D is plotted

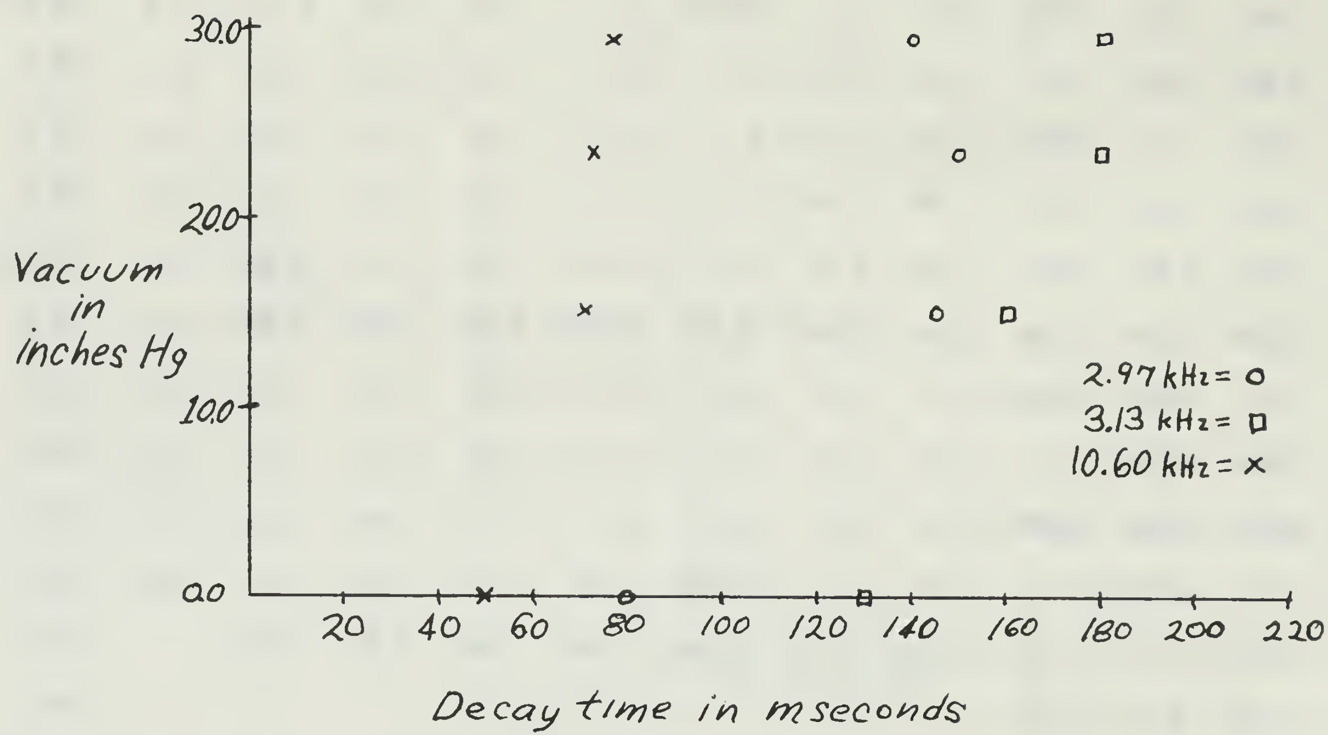


Figure 6 Effect of Vacuum

Frequency in kHz	Depth in cm: 6	8	10	12	15	18	21	24	28	38	48	58
3.00	-	-	-	-	-	-	-	-	60.0	134.0	108.0	140.0
3.18	-	65.0	75.5	65.0	86.0	85.0	80.0	59.0	70.0	121.0	169.0	190.0
3.50	-	-	-	-	-	30.0	-	-	-	143.0	-	190.0
4.10	-	-	-	-	-	-	-	-	100.0	86.8	182.0	150.0
4.60	-	-	-	-	-	-	-	-	-	273.0	115.0	70.0
4.70	-	-	-	-	-	-	-	-	104.0	273.0	-	-
8.31	-	62.0	69.0	59.8	104.0	53.8	55.0	70.3	76.3	119.0	187.0	218.0
8.50	-	52.0	-	-	-	-	-	-	-	-	173.0	140.0
8.90	-	-	-	-	-	-	-	-	-	-	70.0	-
9.10	-	-	-	-	-	-	-	-	115.0	-	86.8	50.0
9.20	-	-	-	-	-	-	-	-	-	75.7	139.0	-
10.60	-	-	-	-	-	67.6	-	-	93.0	56.0	60.5	75.0

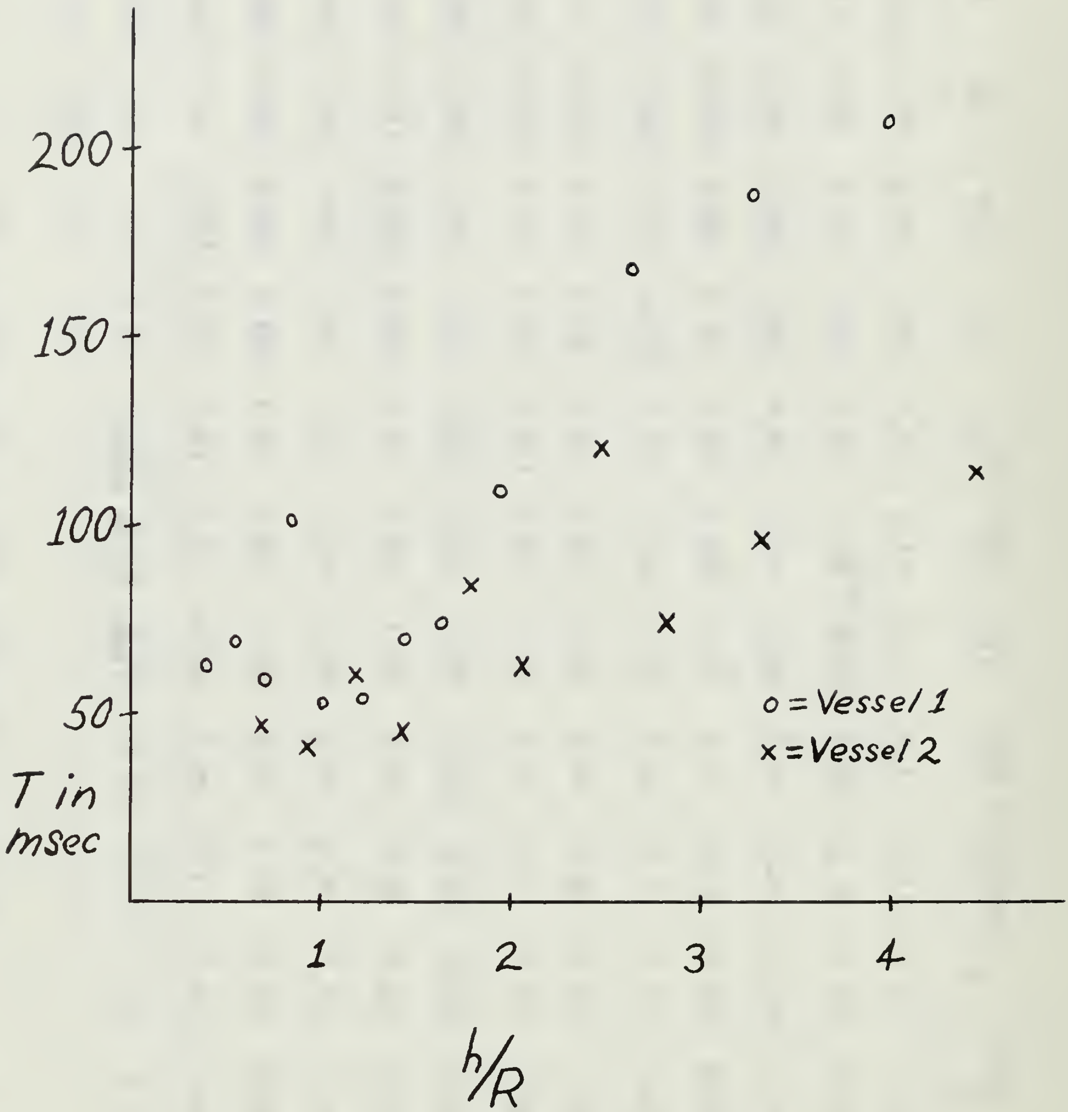
Time, T, in msec

Table II. Measured Decay Time, Vessel No. 1

Frequency in kHz \ Depth in cm	6	8	10	12	15	18	21	24	28	38	48	58
6.11	52.9	36.4	33.0	54.3	73.5	47.8	57.0	74.0	78.0	79.0	135.0	59.0
6.39	57.3	45.1	45.2	52.1	81.4	41.2	53.8	60.0	70.0	65.8	81.6	61.0
6.77	47.7	40.8	59.0	75.0	93.0	41.8	86.8	70.2	53.8	56.5	56.4	80.4
7.33	53.8	28.8	42.4	79.5	73.5	128.0	184.0	76.2	85.8	82.0	139.0	71.0
7.53	89.5	37.3	21.8	60.0	76.3	128.0	205.0	125.0	152.0	110.0	78.0	69.0
8.05	52.1	23.5	34.5	40.0	65.0	129.5	211.0	65.0	159.0	118.0	73.0	65.0
8.31	48.6	42.5	59.2	45.0	84.0	58.2	142.0	74.0	94.5	115.5	86.8	-
14.59	31.3	28.6	38.5	43.4	37.4	40.8	143.0	32.0	66.0	395.0	118.0	82.0
14.75	34.0	56.4	48.6	37.3	30.4	22.6	61.0	60.0	54.8	447.0	191.0	87.0
15.03	42.3	65.0	56.5	41.8	48.6	64.2	61.5	56.0	60.8	45.0	140.0	252.0
15.11	46.0	60.0	57.2	27.8	57.2	34.0	56.4	51.0	87.3	48.0	124.0	265.0
15.68	39.1	48.7	33.0	26.5	38.4	26.0	65.0	53.0	121.0	61.0	115.0	140.0

Time, T, in msec

Table III. Measured Decay Time, Vessel No. 2



Decay times measured at 8.31 kHz

Figure 7 Effect of h/R on Decay time, T

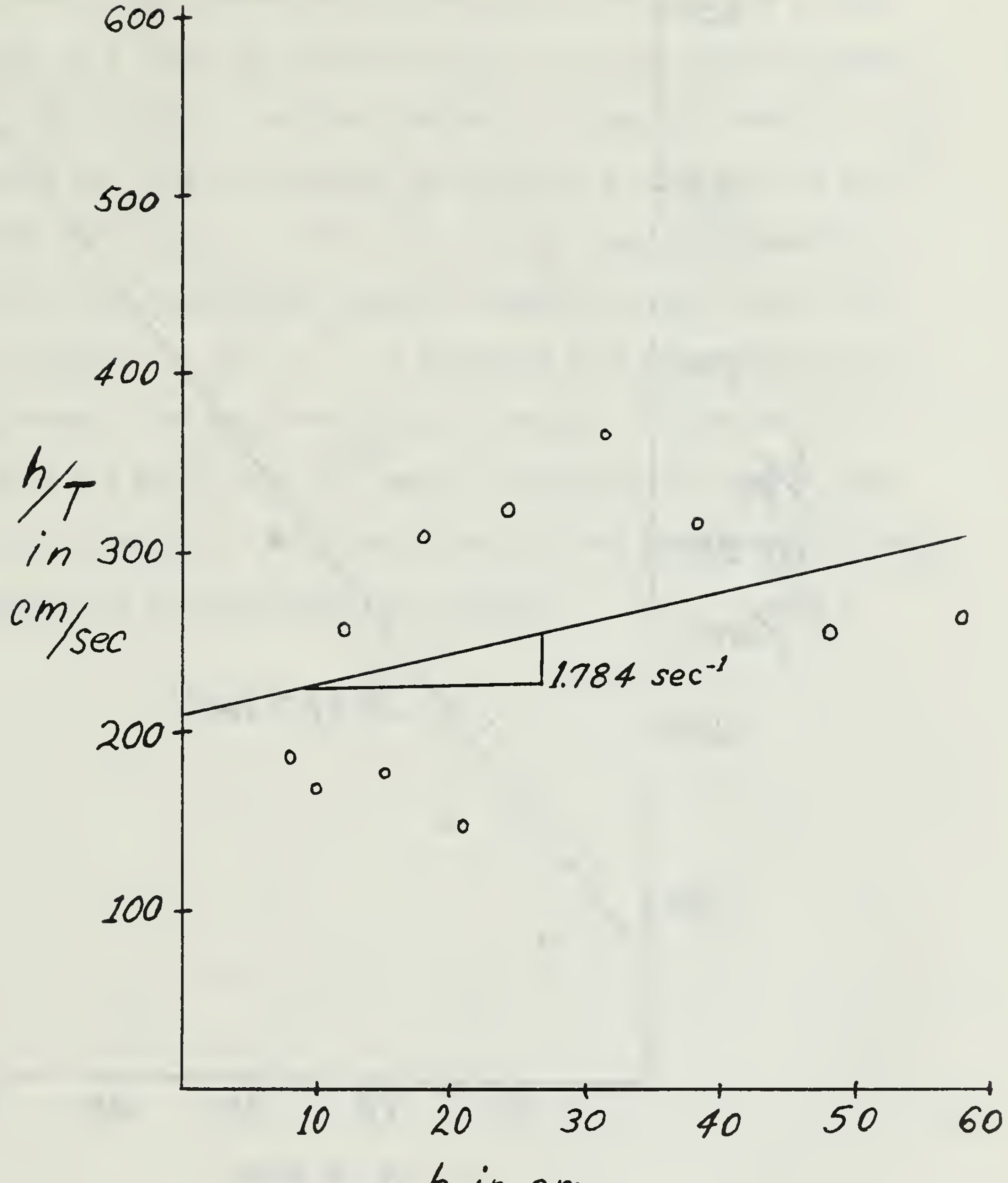


Figure 8 Measurements Taken in Vessel No. 1 at 8.3 kHz as a Function of h

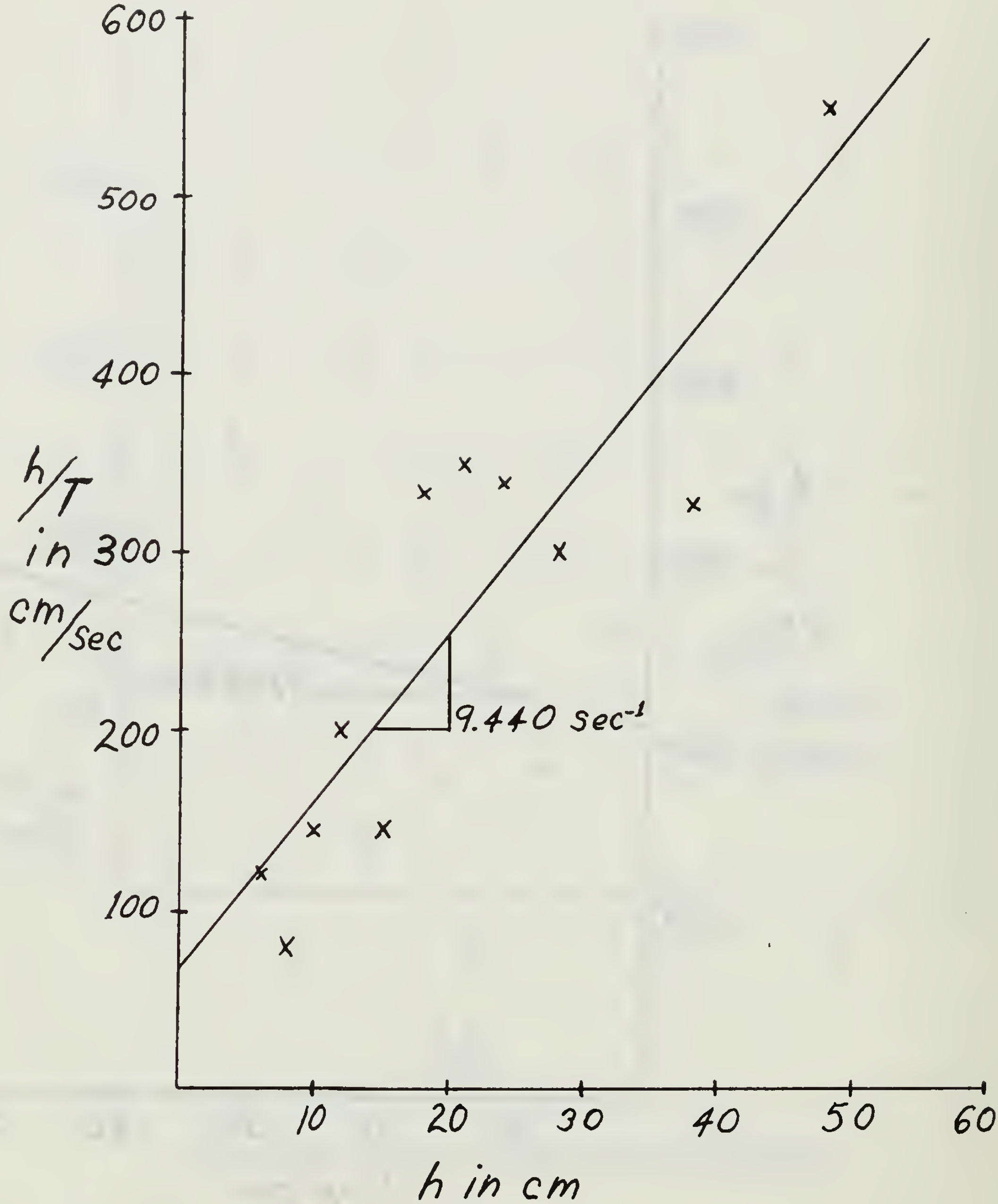


Figure 9 Measurements Taken in Vessel No. 2 at 8.3 kHz as a Function of h

versus 1/R for the same frequency and different vessels the line connecting the data points intercepts the ordinate at the value of D that corresponds to a vessel of infinte radius, i.e., the temporal damping factor of the liquid, κ , at that frequency. Figure 10 shows this plot. By extending the line to intercept the ordinate a value of -9.2 $(sec)^{-1}$ is obtained for κ or -6.1×10^{-3} m^{-1} for the spatial attenuation. The fact that our experimental Kappa is negative rather than a small positive number ($7. \times 10^{-6}$ m^{-1} in reference [9]) is probably due to the low average Q values, about 2660, in vessel 1 and the very low average Q at 8.3 kHz, 1,870 , of vessel 2, which would cause a large error in the value of D. Only the highest Q's can be expected to yield the correct value of the attenuation constant.

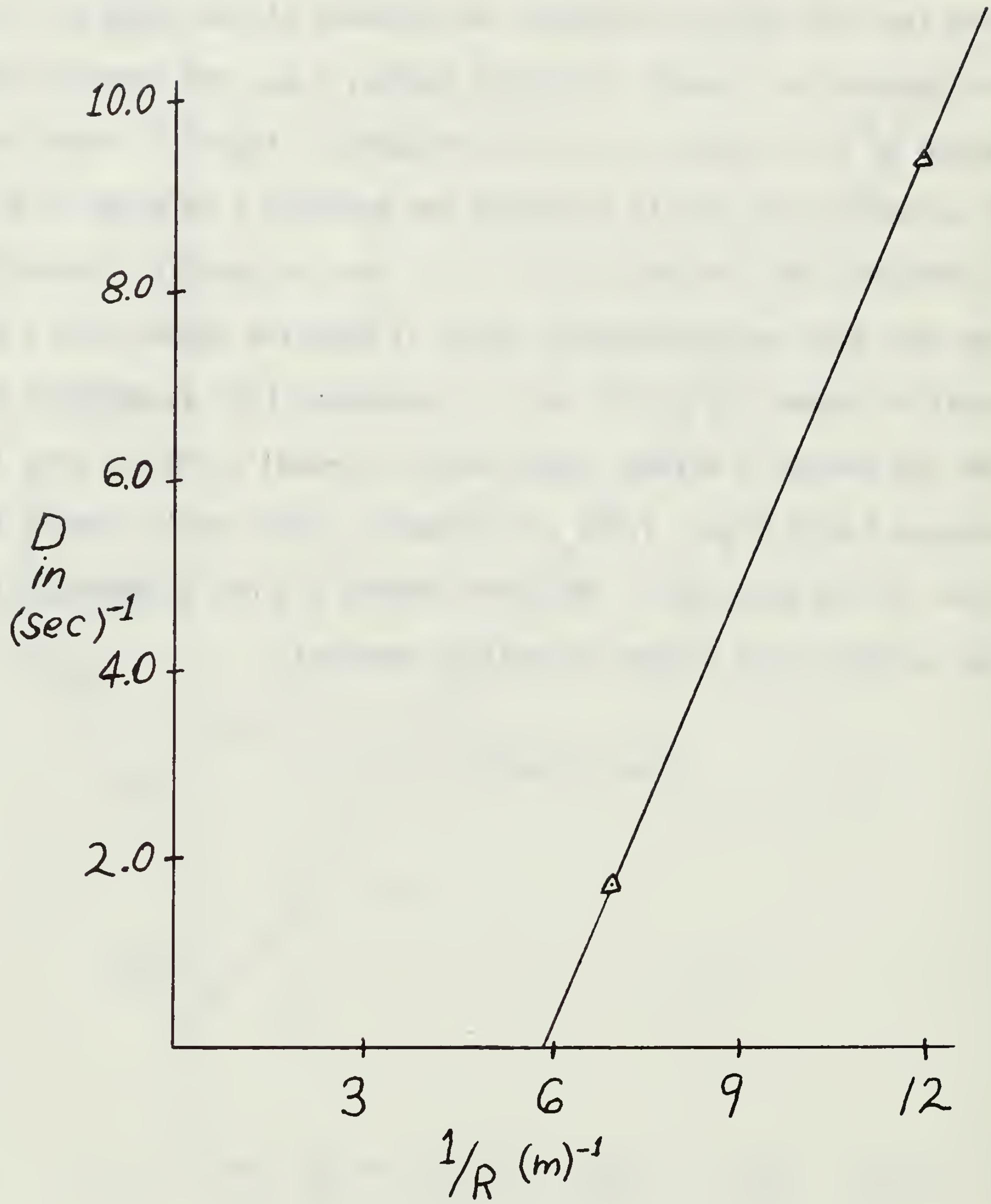


Figure 10 Graphical Determination of Kappa

V. DISCUSSION

Reviewing the requirements for reasonable accuracy, the walls of the vessels should be the same, and the ratio of radius to wall thickness should be at least 200. In addition, the need to excite pure radial modes which are resonant at approximately the same frequency, leads to the specification of the desirable ratio of the vessel radii. That is, the preferable ratio of the radii should be very nearly the same as the ratios of the zeroes of the zeroth order Bessel function.

Early in this investigation, and for a period of several weeks, attempts including catalog searches, letters requesting price quotations, and telephone conversations with manufacturers were made to obtain a pair of vessels completely suited to this investigation. No manufacturer was found who would even consider making the large vessel of fused silica. Only a few were interested in supplying Pyrex vessels of custom specifications. Thus, of the two vessels used in this investigation, No. 1 is a stock item and No. 2 was made on special order.

Because of the difficulties outlined in the foregoing paragraph, each of the requirements concerning the vessel was violated to some degree. The radius to wall thickness ratios were 15 and 23 rather than the desired 200 or greater. The walls in vessel No. 1 were nearly three times as thick as those of vessel No. 2. The desirable ratio of radii of 2.3 could not be obtained either, the actual ratio being approximately 1.7.

For an accurate determination of the absorption coefficient one must use a high Q system. The Q is related to how resonant a system is and for a given frequency longer decay times and higher Q's are directly related. Neither vessel was operating in a high Q mode, as has been shown previously.

The fact that pure radial modes were not excited is completely supported by the experimental evidence shown in Table II and III and in Figures 7, 8, and 9. The measured decay times fluctuate erratically with variation in the depth of the water. For pure radial modes any change in decay time would have a definite observable trend and would not be a strong function of h. Since the pure radial modes have not been strongly excited the modes used to measure the absorption coefficient did have a strong dependence on h and viscous losses were much greater than anticipated.

VI. CONCLUSIONS

The following conclusions can be drawn from the previous sections:

1. In order to optimize the Q of the resonant system for use in low frequency sound attenuation studies:
   a. Evacuation of the area surrounding the cylinder is beneficial.
   b. An h/R ratio of more than unity — preferably more than two — is beneficial.
   c. The radius to wall thickness ratio of Pyrex cylinder containers should be much greater than fifteen.
2. Pure radial modes, desirable to minimize wall shear losses, have not been produced in this investigation in spite of a persistent search for a means to excite them. A thinner-walled vessel would be a means to prevent coupling to other modes, thereby isolating the radial modes as well as increasing the Q of all modes.

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13. ABSTRACT

A pair of large cylindrical Pyrex vessels were water-filled to various heights and the acoustic damping constants and resonant frequencies determined from 3 to 15 kHz. Evacuation of a chamber surrounding the cylinder and increase of the ratio of water height to vessel radius, h/R , greater than unity are both shown to reduce the relative ambient loss of the resonant system. Although the highest Q (20,800) is much less than attained with spherical vessels, it is believed that with thinner walled vessels the extrapolation method of accounting for the influence of the boundaries will permit laboratory measurement of the low frequency attenuation of sound in sea water.

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